# Probing Landau quantization with the presence of insulator–quantum Hall transition in a GaAs two-dimensional electron system

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**Abstract**
Magneto-transport measurements are performed on the two-dimensional electron system (2DES) in an AlGaAs/GaAs heterostructure. By increasing the magnetic field perpendicular to the 2DES, magneto-resistivity oscillations due to Landau quantization can be identified just near the direct insulator–quantum Hall (I–QH) transition. However, different mobilities are obtained from the oscillations and transition point. Our study shows that the direct I–QH transition does not always correspond to the onset of strong localization.

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

The insulator to quantum Hall (I–QH) transition in a two-dimensional electron system (2DES) at low perpendicular magnetic fields $B$ has attracted much attention [1–11]. Theoretically, the direct I–QH transition from the insulator to an integer QH state of $\nu \neq 1$ is forbidden in an infinite, non-interacting 2DES with an arbitrary amount of disorder, where $\nu$ is the Landau-level filling factor [1–3]. In such a system, the only allowed state at $B = 0$ is the insulating one, and the

2DES undergoes the I–QH transition to enter the $\nu = 1$ QH state [12, 13]. Realistically, however, only systems of finite sizes are available, and the effects of the electron–electron (e–e) interaction are significant in some 2DESs [4, 5, 14–18]. As a result, the 2DESs may experience the direct I–QH transition from the low-field insulator to QH states of higher filling factors [2–4, 8, 16–18]. Such a transition can be related to the zero-field metal–insulator transition, in which e–e interaction cannot be ignored [4]. Given that most 2DESs show metallic behaviour at $B = 0$, the investigation of the direct I–QH

transition at low $B$ should be conducted in low-mobility 2DESs [1, 12].

The mechanisms for the direct I–QH transition are still under debate [5–7, 11, 17, 18]. Huckestein [5] argued that such a transition is a crossover from weak localization to Landau quantization rather than a phase transition. Therefore the observed transition or crossing point is not a critical point. According to Huckestein's argument, such a point should occur as the product

$$\mu B = 1. \qquad (1)$$

Here $\mu$ is the mobility such that the strong localization due to high-field Landau quantization becomes important when the product $\mu B$, which equals the ratio of Landau-level spacing to broadening, is large enough. To be a measure for Landau quantization, $\mu$ should be the quantum mobility. Because the strong localization is believed to be important to the QH liquid, it seems natural that a 2DES undergoes the direct I–QH transition at $\mu B = 1$ as we increase the perpendicular magnetic field. However, experimental evidence of quantum phase transition has been observed near the transition point [8]. In addition, the existence of Landau quantization in the low-field insulator indicates that its onset may be irrelevant to such a transition [9, 10]. In fact, Landau quantization could be unimportant to the crossover because its feature is absent near the crossing point in some reports [14, 15]. Corrections based on the e–e interaction [14–16, 18, 19] and low-field Landau quantization effects [9–11] are discussed in the literature. On the other hand, magneto-oscillations due to Landau quantization appear just near the direct I–QH transition with increasing $B$ in some reports [2, 3, 8]. Huckestein's argument seems correct if we identify the onset of Landau quantization by the appearance of magneto-oscillations. The observations of

$$\rho_{xy}/\rho_{xx} \approx 1, \qquad (2)$$

near the transition points [2, 3] are also consistent with Huckestein's argument because $\rho_{xy}/\rho_{xx} = \mu B$ in the Drude model if the transport and quantum mobilities are the same. Here $\rho_{xx}$ and $\rho_{xy}$ are the longitudinal and Hall resistivities, respectively. To understand the direct I–QH transition, therefore, we shall re-examine the 2DESs where Landau quantization induces oscillations just near the transition point occurring as equation (2) becomes valid with increasing $B$.

In this study, we report a magneto-transport investigation on the 2DES in an AlGaAs/GaAs heterostructure. With increasing magnetic field $B$, amplitudes of resistivity oscillations $\Delta\rho_{xx}$ following the Shubnikov–de Haas (SdH) formula [20–24]

$$\Delta\rho_{xx} \propto \frac{\chi}{\sinh\chi} \exp\left(\frac{-\pi}{\mu B}\right) \qquad (3)$$

with $\chi = 4\pi^3 k m^* T / h e B$ can be identified just as the 2DES undergoes the direct I–QH transition. Here $T$ is the temperature, $k$, $h$, $e$, and $m^*$ are denoted as Boltzmann constant, Plank constant, electron charge, and effective mass, respectively. The oscillations are features of Landau quantization, so it seems that the observed direct transition occurs near the onset of Landau quantization just as suggested
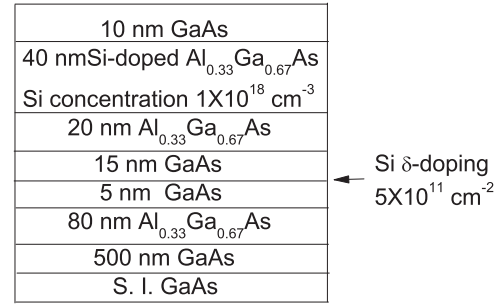


**Figure 1.** A Schematic diagram showing the sample structure.

by Huckestein. In addition, equation (2) is valid at the transition point. However, different mobilities should be introduced just as in references [14, 15] because $\mu B$ is much smaller than 1 at the crossing point. One is for the direct I–QH transition and the other is for Landau quantization. Therefore, corrections to Huckestein's argument should be taken into account even when the onset of Landau quantization can be approximated by the transition point where equation (2) is valid.

The experimental conditions are described in section 2, and the investigations on mobilities near the I–QH transition are discussed in section 3. Effects due to electron–electron interaction, electron–phonon scattering and disorder-enhanced electron–electron scattering are mentioned in section 4, and the conclusion is made in section 5.

## 2. Experimental details

The sample (LM4646) used in this study is an AlGaAs/GaAs heterostructure. Figure 1 shows its structure, where some Si atoms are doped in the 20 nm-wide GaAs quantum well to serve as the scattering sources. It is known that we can suppress the mobility to probe the integer quantum Hall effect by deliberately introducing some scattering sources in the quantum wells [3, 9, 10]. The sample is made into the Hall pattern with the channel width 80 $\mu$m by standard optical lithography, and AuGeNi alloy is annealed at 450 °C to fabricate the ohmic contacts. The magneto-transport measurements are performed in a top-loading He$^3$ system with the superconducting magnet.

## 3. Insulator–quantum Hall transition and mobility analysis

Figure 2 shows the curves of the longitudinal resistivity $\rho_{xx}(B)$ at different temperatures and Hall resistivity $\rho_{xy}(B)$ at the temperature $T = 4$ K under a low-frequency AC driving current of 40 nA. At low $B$, the 2DES behaves as an insulator such that $\rho_{xx}$ increases with decreasing $T$. The insulator is terminated at $B = 3.5$ T $\equiv B_c$, and $\rho_{xx}$ decreases with decreasing $T$ at $B > B_c$. Therefore, $B_c$ is the transition point. The filling factor $\nu \sim 8$ at $B_c$, and oscillations periodic in $1/B$ are observed when the sample behaves as a QH liquid at $B > B_c$. From the oscillating period in $1/B$, the carrier
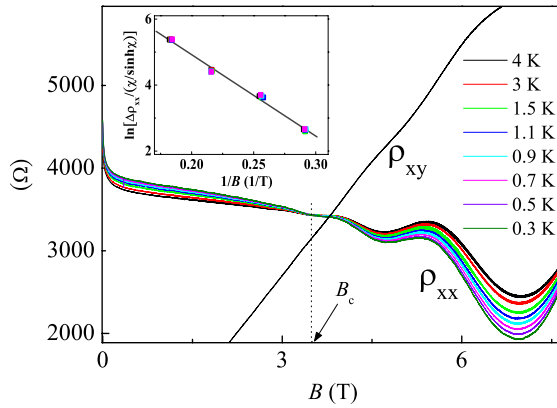
**Figure 2.** Longitudinal and Hall resistivity as a function of magnetic field ($B$) at various temperatures $T$. The dotted line indicates the transition point $B_c$. The inset shows $\ln(\Delta\rho_{xx}/(\chi/\sinh\chi))$ as a function of $1/B$ at $T = 0.7, 0.9, 1.1, 1.3, 1.5, 3$ and $4$ K, respectively.

concentration $n = 6.8 \times 10^{15}$ m$^{-2}$. We can see in figure 2, that an SdH dip appears as $B \sim B_c$, so the observed I–QH transition at $B_c$ is a direct one [2, 3, 5]. In figure 2, magneto-oscillations cannot be observed at low $B$ until we increase the magnetic field to about $B = B_c$. Since such oscillations are due to Landau quantization, the 2DES provides us an opportunity to probe the direct I–QH transition which occurs as Landau quantization can just be identified. In addition, we can see that $\rho_{xx} = 3.4$ k$\Omega \approx \rho_{xy} = 3.1$ k$\Omega = \frac{B}{ne}$ at $B_c$ at $T = 4$ K although the Hall slope is weakly $T$-dependent. So the observed transition occurs as $\rho_{xx}/\rho_{xy} \approx 1$, which seems to be consistent with Huckestein's argument. The low-field oscillations are expected to follow equation (3), the SdH formula. To analyze the mobility from equation (3), we note that $\ln(\Delta\rho_{xx}/(\chi/\sinh\chi)) = \text{const} - \pi/(\mu B)$. We can see from the inset to figure 2 that the data of $\ln(\Delta\rho_{xx}/(\chi/\sinh\chi)) - 1/B$ at different temperatures collapse well into a single straight line when we take $m^* = 0.067m_0$ as the expected value in a GaAs 2DES. From the slope of $\ln(\Delta\rho_{xx}/(\chi/\sinh\chi)) - 1/B$, the quantum mobility $\mu = 0.13$ m$^2$ V$^{-1}$ s$^{-1}$. Therefore, we can obtain the product $\mu B = 0.46$ at the transition point $B = B_c$. Such a product deviates much from 1, and thus our result is inconsistent with Huckestein's argument although the direct I–QH transition occurs just as the magneto-oscillations due to Landau quantization can be observed under equation (2). Since the conventional SdH formula is based on Landau quantization without considering strong localization, the fact that the experimental data shown in the inset to figure 2 can be well fitted to equation (3) suggests for $B \leqslant 5.4$ T strong localization may not be significant in our system.

It is known that Landau quantization can result in magneto-oscillations as the product $\mu B < 1$ [25]. Therefore, the appearance of magneto-oscillations near $B_c$ does not indicate that the transition occurs just as equation (1) is valid. While numerical studies show that such transitions can occur just as $\mu B \approx 1$ in a non-interacting 2DES, Landau quantization can induce magneto-oscillations at $\mu B < 1$
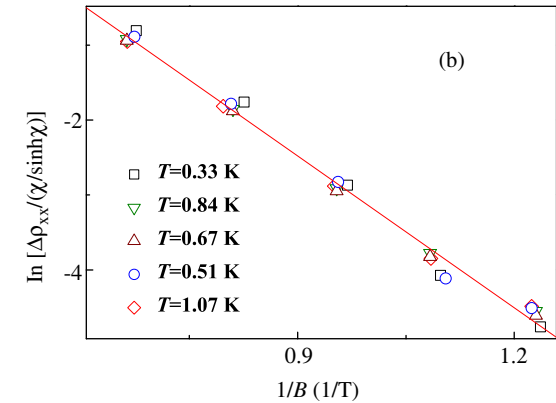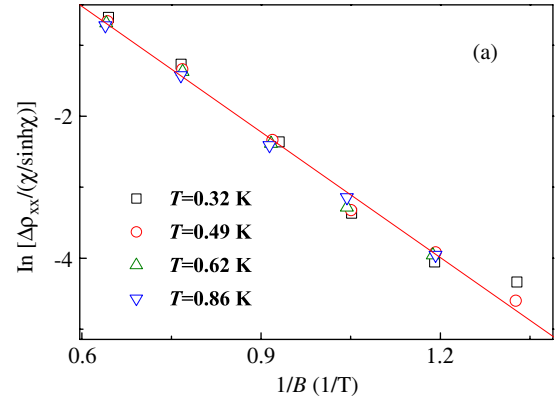


**Figure 3.** $\ln(\Delta\rho_{xx}/(\chi/\sinh\chi))$ as a function of $1/B$ at (a) $V_g = +0.15$ V and (b) $V_g = 0$ at different temperatures $T$.

where such a 2DES is an insulator [11]. The coexistence of magneto-oscillations and insulating behaviour can be explained by the percolation theory [26, 27]. We note that Huckestein considered only a single mobility based on the Drude model, but another mobility $\mu'$ has been introduced in [14–16, 18]. The mobility $\mu$ corresponds to the quantum mobility while $\mu'$ can be related to the transport mobility although renormalization effects may be important [16]. The direct I–QH transition should occur as $\mu'B = 1$, and thus we obtain $\mu' = 1/B_c = 0.29$ m$^2$ V$^{-1}$ s$^{-1}$ $\approx 2.2\mu$. Therefore, different mobilities should still be taken into account even as Landau quantization can be identified near $B_c$ with increasing $B$.

To further check Landau quantization near direct I–QH transitions, we re-examine the data published in our previous report [8]. In that report, we also investigated direct I–QH transitions, near which magneto-oscillations can be identified, at low magnetic fields in a gated 2DES. Magneto-oscillations can be observed as the filling factor $\nu \sim 10$ in such a 2DES when the gate voltage $V_g = +0.15$ and $0$ V, and we can apply equation (3) to analyze the quantum mobility after the appearance of I–QH transitions. Figures 3(a) and (b) show the curves of $\ln(\Delta\rho_{xx}/(\chi/\sinh\chi)) - 1/B$ at these two gate voltages, and the slopes yield $\mu = 0.53$ and $0.47$ m$^2$ V$^{-1}$ s$^{-1}$ at $V_g = +0.15$ and $0$ V, respectively. On the other hand, the transition points yield $\mu' = 1.9$ and $1.7$ m$^2$ V$^{-1}$ s$^{-1}$ under these two gate voltages. The quantum mobility $\mu$ is much lower than the mobility $\mu'$ obtained from the transition point.
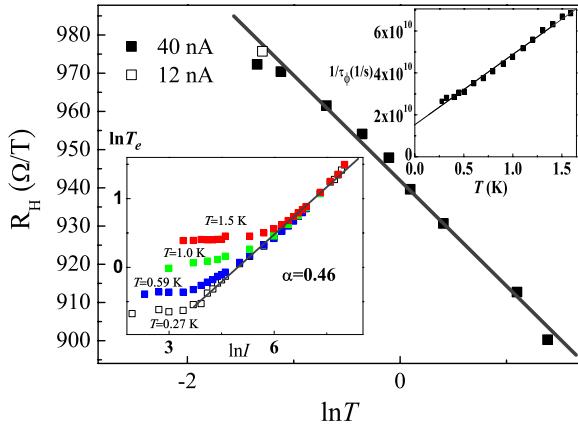
**Figure 4.** Hall slope as a function of $T$. The black squares represents the result obtained at $I = 40$ nA while the open square corresponds to the data obtained at $I = 12$ nA. The straight line corresponds to the best linear fit at $T = 0.5$–4 K. The lower inset shows the logarithm of electron effective temperature $T_e$ as a function of logarithm of current $I$ determined from the zero-field resistivity $\rho_{xx}$ at different lattice temperature $T$. The best linear fit corresponds to $T_e \propto I^\alpha$ with $\alpha = 0.46$. The upper inset shows the inverse of phase coherence time $1/\tau_\phi$ as a function of $T$.

Therefore, different mobilities should also be introduced to understand the direct I–QH transitions.

In Huckestein's argument, the direct I–QH transition separates the weak-localization regime from the QH liquid due to the strong localization under Landau quantization. At low $B$, however, either Landau quantization or the quantum Hall effect can be irrelevant to the strong localization effect [20, 23, 24, 28–30]. The onset of magneto-oscillations following equation (3) near the transition field $B_c$, in fact, does not indicate the importance of the strong localization to the direct I–QH transition because equation (3) can hold without any localization effect [20, 31]. Huckestein's argument is valid only if the onsets of both the strong localization and Landau quantization are at $\mu B \approx \mu' B \approx \rho_{xy}/\rho_{xx} \approx 1$. Our study shows that the direct I–QH transition does not always indicate the onset of strong localization even when Landau quantization can be identified near the transition point with increasing $B$.

## 4. Discussion

Corrections based on the e–e interaction effect [14–16] have been taken into account for the direct I–QH transition when the magnetic field is too weak to induce the high-field strong localization effect. In our study, as shown later, there exists evidence for e–e interaction and scattering although semiclassical and electron–phonon effects should be also considered.

### 4.1. T-dependent Hall slope and e–e interaction

The e–e interaction effect can modify the 2D density of states near the Fermi level, giving rise to a logarithmic $T$-dependent Hall slope of a 2DES [32]. As shown in figure 4, the Hall slope is logarithmic $T$-dependent at $T = 0.5$–4 K in the 2DES in

sample LM4646. Since the carrier density determined from the oscillations in $\rho_{xx}$ remains constant over the same temperature range, the observed logarithmic $T$-dependent Hall slope can only be ascribed to e–e interaction effect within our system. This experimental evidence for e–e interactions suggests that such effects could be important to the observed I–QH transition in our system. The parabolic negative magneto-resistance, however, is not apparent at $\mu B < 1$ in figure 2 although it is also expected under the e–e corrections [14]. In addition, we note that the magneto-oscillations are absent at $B_c$ in [14] and [15] while they appear near the transition point in our study and in [2, 3]. In different 2D systems, therefore, it is possible that the dominant effects and/or parameters are not the same at low fields [14, 15, 33].

### 4.2. Electron effective temperature

We can see from figure 4 that the Hall slope under a current $I = 40$ nA deviates somewhat from the expected logarithmic $T$ dependence at the lowest temperature. To understand the mechanism for the deviation, we note that $\rho_{xx}$ at $B = 0$ is $I$-dependent with increasing current. Here $\rho_{xx}(B = 0)$ represents the value of $\rho_{xx}$ at zero magnetic field. The $I$-dependence indicates the existence of the current heating, under which the electron effective temperature $T_e$ is higher than the lattice temperature $T$ [34]. Therefore, effects due to electron–phonon interaction could be important in our study for electrons to transfer the extra energy to the lattice, which can induce the deviation of the Hall slope at low $T$. The temperature dependence of $\rho_{xx}$ at $B = 0$ can be used as a self thermometer to determine $T_e$ as follows. Under a low-current without inducing electron heating, $T_e$ should equal the lattice temperature $T$ and the dependence of $\rho_{xx}$ at $B = 0$ with respect to $T_e = T$ can be obtained by direct measurements. Because $\rho_{xx}$ at zero magnetic field is a decreasing function of $T$ (or $T_e$) under a low enough current in our study, the value of $\rho_{xx}$ and $T_e$ is in one–one correspondence at $B = 0$. Then at a fixed lattice temperature $T$, we can raise $T_e$ by increasing the current $I$ and determine $T_e$ from such a correspondence. In this way, the $I$-dependence of $T_e$ at zero field is determined, and the lower inset shows the relation between $T_e$ and $I$ at different lattice temperature when $B = 0$. We can see from such an inset that the zero-field resistivity data shows $T_e \propto I^\alpha$ with the exponent $\alpha = 0.46 \approx 0.5$, which is expected under the electron–phonon interaction [35]. The current and temperature dependences of the Hall slope yields $\alpha = 0.53$, which is also close to 0.5. Actually the low-field regime is unstable in the global phase diagram of the quantum Hall effect [1], and more studies are necessary to clarify the dominant effects and/or parameters at low magnetic fields [11, 14–16, 18, 19, 21–24, 28–30].

### 4.3. Phase coherence time analysis and e–e scattering

By decreasing the current to $I = 12$ nA, as indicated by the open square in figure 4, the deviation on the logarithmic $T$-dependence of the Hall slope at low $T$ can be removed. In addition, we note that the direct I–QH transition at $\mu B = 1$ can still be related to the e–e interaction effect when corrections

to the negative magneto-resistance are taken into account. Moreover, the linear $T$-dependence of the inverse of the phase coherence time $\tau_\phi$ in the upper inset to figure 4 indicates the scattering due to the e–e interaction while the nonzero intercept shows the zero-temperature dephasing [37]. The slope of $1/\tau_\phi$–$T$ equals $3.45 \times 10^{10}$ s$^{-1}$ K$^{-1}$, which is a reasonable value under the e–e scattering [32]. The phase coherence time $\tau_\phi$ is obtained by fitting our data to the low-field equation [36]

$$\Delta\sigma_{xx}(B) = \frac{-e^2}{\pi h}\left[\Psi\left(\frac{1}{2} + \frac{B_0}{B}\right) - \Psi\left(\frac{1}{2} + \frac{B_\phi}{B}\right)\right], \quad (4)$$

where $\Psi$ is the digamma function and $B_0$ and $B_\phi$ correspond to transport and phase coherence rates, respectively [32]. Therefore, the direct I–QH transition in our study could be dominated by the e–e interaction effect rather than the onset of Landau quantization although different mechanisms should be introduced to understand the details. In our study, both $\mu$ and $\mu'$ remain the same after decreasing the driving current, which also indicates that the current heating and/or electron–phonon interaction is irrelevant to the difference between these two mobilities.

## 5. Conclusion

In conclusion, we investigate Landau quantization and the direct I–QH transition in the two-dimensional electron system in an AlGaAs/GaAs heterostructure. Our study shows that such a transition does not occur as $\mu B = 1$ even when Landau quantization can be identified near the transition point by the appearance of magneto-oscillations as $\rho_{xy}/\rho_{xx} \approx 1$. Therefore, our study supports that different mobilities should be introduced for the direct I–QH transition and Landau quantization. The temperature dependences of the Hall slope and dephasing time indicate the importance of the effects of the e–e interaction to the direct I–QH transition although different mechanisms should be considered for the details of such a transition. The appearance of Landau quantization or direct I–QH transition, in fact, does not always correspond to the onset of the strong localization effect giving rise to quantum Hall liquids.

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## References

[1] Kivelson S, Lee D-H and Zhang S C 1992 *Phys. Rev.* B **46** 2223
[2] Song S-H, Shahar D, Tsui D C, Xie Y H and Monroe D 1997 *Phys. Rev. Lett.* **78** 2200
[3] Lee C H, Chang Y H, Suen Y W and Lin H H 1998 *Phys. Rev.* B **58** 10629
[4] Hanein Y, Nenadovic N, Shahar D, Shtrikman H, Yoon J, Li C C and Tsui D C 1999 *Nature* **400** 735
[5] Huckestein B 2000 *Phys. Rev. Lett.* **84** 3141
[6] Liu D Z, Xie X C and Niu Q 1996 *Phys. Rev. Lett.* **76** 975
[7] Sheng D N and Weng Z Y 1998 *Phys. Rev. Lett.* **80** 580
[8] Huang C F, Chang Y H, Lee C H, Chou H T, Yeh H D, Liang C-T, Chen Y F, Lin H H, Cheng H H and Hwang G J 2002 *Phys. Rev.* B **65** 045303
[9] Kim G-H, Liang C-T, Huang C F, Nicholls J T, Ritchie D A, Kim P S, Oh C H, Juang J R and Chang Y H 2004 *Phys. Rev.* B **69** 073311
[10] Huang T-Y, Juang J R, Huang C F, Kim G-H, Huang C P, Liang C-T, Chang Y H, Chen Y F, Lee Y and Ritchie D A 2004 *Physica* E **22** 240
[11] Nita M, Aldea A and Zittartz J 2000 *Phys. Rev.* B **62** 15367
[12] Jiang H W, Johnson C E, Wang K L and Hannah S T 1993 *Phys. Rev. Lett.* **71** 1439
[13] Hughes R J F, Nicholls J T, Frost J E F, Linfield E H, Pepper M, Ford C J B, Ritchie D A, Jones G A C, Kogan E and Kaveh M 1994 *J. Phys.: Condens. Matter* **6** 4763
[14] Li L, Proskuryakov Y Y, Savchenko A K, Linfield E H and Ritchie D A 2003 *Phys. Rev. Lett.* **90** 076802
[15] Cho H-I, Gusev G M, Kvon Z D, Renard V T, Lee J H and Portal J C 2005 *Phys. Rev.* B **71** 245323
[16] Minkov G M, Germanenko A V, Rut O E, Sherstobitov A A, Larionova V A, Bakarov A K and Zvonkov B N 2006 *Phys. Rev.* B **74** 045314
[17] Minkov G M, Rut O E, Germanenko A V, Sherstobitov A A, Zvonkov B N, Uskova E A and Birukov A A 2001 *Phys. Rev.* B **65** 235322
[18] Arapov Yu G, Yakunin M V, Gudina S V, Harus G I, Neverov V N, Shelushinina N G, Podgornyh S M, Uskova E A and Zvonkov B N 2007 *AIP Conf. Proc.* **893** 647
[19] Kaminskii V E 2003 *Phys. Rev.* B **67** 085201
[20] Coleridge P T 1990 *Semicond. Sci. Technol.* **5** 961 and references therein
[21] Martin G W, Maslov D L and Reizer M Y 2003 *Phys. Rev.* B **68** 241309
[22] Adamov Y, Gornyi I V and Mirlin A D 2006 *Phys. Rev.* B **73** 045426
[23] Hang D R, Huang C F, Zhang Y W, Yeh H D, Hsiao J C and Peng H L 2007 *Solid State Commun.* **141** 17
[24] Chen J-H, Hang D R, Huang C F, Huang T-Y, Lin J-Y, Lo S H, Hsiao J C, Lin M-G, Simmons M Y, Ritchie D A and Liang C-T 2007 *J. Korean Phys. Soc.* **50** 776
[25] Huang T-Y, Liang C-T, Kim G-H, Huang C F, Huang C P, Lin J Y, Goan H-S and Ritchie D A 2006 *Preprint* cond-mat/0601490 references in
[26] Shimshoni E, Auerbach A and Kapitulnik A 1998 *Phys. Rev. Lett.* **80** 3352
[27] Trugman S A 1983 *Phys. Rev.* B **27** 7539
[28] Coleridge P T 1994 *Phys. Rev. Lett.* **72** 3917
[29] Kramer T, Bracher C and Kleber M 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** 21
[30] Kramer T 2006 *Int. J. Mod. Phys.* B **20** 1243
[31] Fogler M M, Dobin A Yu, Perel V I and Shklovskii B I 1997 *Phys. Rev.* B **56** 6823
[32] For example see, Simmons M Y, Hamilton A R, Pepper M, Linfield E H, Rose P D and Ritchie D A 2000 *Phys. Rev. Lett.* **84** 2489 and references therein
[33] Gusev G M, Basmaji P, Kvon Z D, Litvin L V, Nastaushev Yu V and Toropov A I 1994 *J. Phys.: Condens. Matter* **6** 73
[34] Wennberg A K M, Ytterboe S N, Gould C M, Bozler H M, Klem J and Morkoc H 1986 *Phys. Rev.* B **34** 4409
[35] For example see, Scherer H, Schweitzer L, Ahlers F J, Bliek L, Losch R and Schlaap W 1996 *Semicond. Sci Technol.* **10** 959 and references therein
[36] Hikami S, Larkin A I and Nagaoka Y 1980 *Prog. Theor. Phys.* **63** 707
[37] For a review please see, Lin J J and Bird J P 2002 *J. Phys.: Condens. Matter* **14** R501